

BACCALAURÉAT GÉNÉRAL ET TECHNOLOGIQUE
ÉPREUVE SPÉCIFIQUE DES SECTIONS EUROPÉENNES
MATHEMATIQUES – ANGLAIS

SUJET 9 – GOLDEN RATIO

Thème : Séquences

Ce sujet comporte 1 page. L'usage de la calculatrice est autorisé.

Fibonacci (1175-1250) is one of the most famous names in mathematics. This would come as a surprise to Leonardo Pisano, the mathematician we now know by that name. And he might have been equally surprised that he has been immortalised in the famous sequence – 0, 1, 1, 2, 3, 5, 8, 13, ... – rather than for what is considered his far greater mathematical achievement – helping to popularise our modern number system in the Latin-speaking world.

The numbers of that sequence are famous because we find them in nature, in architecture, in music, in art.

Actually, if you calculate the ratio of two consecutive Fibonacci's numbers, you obtain:

$$\frac{1}{1} = 1, \frac{2}{1} = 2, \frac{3}{2} = 1.5, \frac{5}{3} \approx 1.666 \dots, \frac{8}{5} = 1.6, \frac{13}{8} = 1.625, \dots$$

These calculations are linked to φ which is named the « Golden Ratio ».

The golden ratio is found in the pyramids of Giza and the Parthenon of Athens, the cathedral « Notre dame ». It is also found in paintings such as Leonardo da Vinci's ones.

Source: Extract from <https://plus.maths.org/content/life-and-numbers-fibonacci>

- 1- Read the first paragraph of the text
- 2- What does the text deal with?

Exercise:

- 1- Fibonacci sequence is defined : $u_{n+1} = u_n + u_{n-1}$, with $u_0 = 1$ and $u_1 = 1$
 - a) Calculate, the first ten terms of that sequence.
 - b) Is it an arithmetic sequence? A geometric sequence? (Explain why)
 - c) Work out the ratios of the first terms you have calculated in question 1 as in the text above.
- 2- φ , the golden number is equal to $\frac{1+\sqrt{5}}{2}$. Is there a link with the ratios calculated in question 1.c?
- 3- A golden rectangle is a rectangle in which the lengths are a and b such as $a/b = (a+b)/a = \varphi$
 - a) What is the link with the previous questions?
 - b) We know another number linked to a geometric shape: calculate the length of the diagonal of a square.